Lecture 3

Randomised block designs

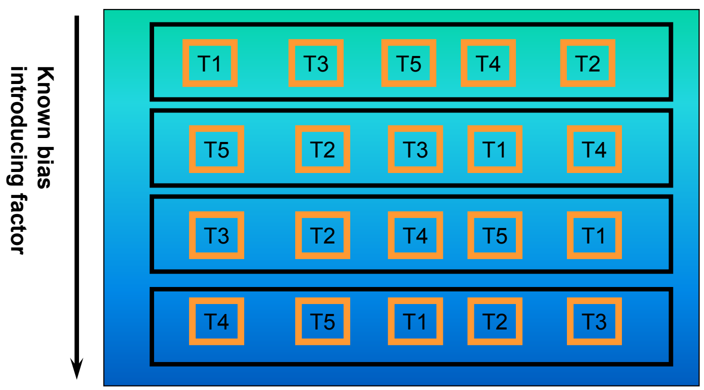
What's expected to be understood

- **Multiple regression in Response Surface Methodology. Dependent and independent variables are numeric. The predicted response is studied.**
- **N- way ANOVA including multiple comparison tests. Dependent variable is numeric, independent variables are categorical. Differences between treatment level means are investigated for N experimental factor, including possible interaction effects between factors. Model parameters are investigated.**
- **In all approaches Completely Randomised Designs are assumed.**

What's next?

Generalisation to more complicated experimental designs: Randomised Block and Latin Squares Designs

Blocking Example



Block effect now removes the effect of this factor.
Thus, fair comparisons among treatments are possible.

Single Replicate RCBD

Design: Complete block layout with each treatment replicated once in each block.

Data:

	Block				
Treatment	1	2	3	...	b
1	y_{11}	y_{12}	y_{13}	...	y_{1b}
2	y_{21}	y_{22}	y_{23}	...	y_{2b}
...	...	...	...	...	...
t	y_{t1}	y_{t2}	y_{t3}	...	y_{tb}

Linear Analysis Model

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \qquad i = 1 \cdots t, \quad j = 1 \cdots b$$

constraints

$$E(y_{ij}) = \mu + \alpha_i + \beta_j = \mu_{ij} \qquad \sum_i \alpha_i = 0, \quad \sum_i \beta_i = 0$$

	Block					
Treatment	1	2	3	...	b	sum
1	μ_{11}	μ_{12}	μ_{13}	...	μ_{1b}	$\mu + \alpha_1$
2	μ_{21}	μ_{22}	μ_{23}	...	μ_{2b}	$\mu + \alpha_2$
...	...	...	...	...	...	
t	μ_{t1}	μ_{t2}	μ_{t3}	...	μ_{tb}	$\mu + \alpha_t$
sum	$\mu + \beta_1$	$\mu + \beta_2$	$\mu + \beta_3$		$\mu + \beta_b$	

RCBD AOV

Source	SSQ	df	MS	F
Treatments	SST	t-1	MST=SST/(t-1)	MST/MSE
Blocks	SSB	b-1	MSB=SSB/(b-1)	MSB/MSE
Error	SSE	(b-1)(t-1)	MSE=SSE/(b-1)(t-1)	
Totals	TSS	bt-1		

Partitioning of the total sums of squares (TSS)

TSS = SST + SSB + SSE

Regression Sums of Squares (SST + SSB)

$$df_{Total} = df_{Treatment} + df_{Block} + df_{Error}$$

Sums of Squares - RCBD

$$TSS = \sum_{i=1}^{t}\sum_{j=1}^{b}(y_{ij} - \bar{y}_{\bullet\bullet})^2$$

$$SST = b\sum_{i=1}^{t}(\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet})^2$$

$$SSB = t\sum_{j=1}^{b}(\bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet})^2$$

$$SSE = \sum_{i=1}^{t}\sum_{j=1}^{b}(y_{ij} - \bar{y}_{i\bullet} - \bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet})^2$$

$$TSS = \sum_{i=1}^{t}\sum_{j=1}^{b} y_{ij}^2 - \frac{\bar{y}_{\bullet\bullet}}{bt}$$

$$SST = \sum_{i=1}^{t}\frac{y_{i\bullet}^2}{b} - \frac{\bar{y}_{\bullet\bullet}}{bt}$$

$$SSB = \sum_{j=1}^{b}\frac{y_{\bullet j}^2}{t} - \frac{\bar{y}_{\bullet\bullet}}{bt}$$

$$SSE = TSS - SST - SSE$$

Expectation under the alternative hypothesis. →

$$E(MSE) = \sigma_\varepsilon^2$$

$$E(MST) = \sigma_\varepsilon^2 + b\theta_T$$

$$E(MSB) = \sigma_\varepsilon^2 + t\theta_B$$

$$\theta_T = \frac{\sum_i \alpha_i^2}{b} \qquad \theta_B = \frac{\sum_j \beta_j^2}{t}$$

Blocking example 1

A scientist was interested in the use of three chemicals and water on their effectiveness in extracting sulfur from Florida soils. The chemicals of interest are:

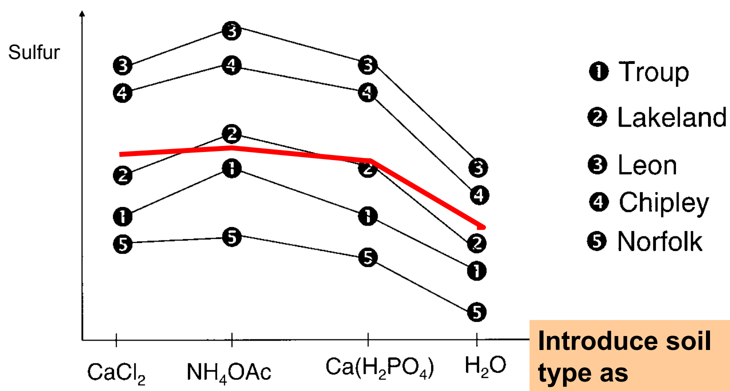
•Calcium Chloride	$CaCl_2$
•Ammonium Acetate	NH_4OAc
•Mono Calcium Phosphate	$Ca(H_2PO_4)_3$
•Water	H_2O

Five soils were chosen for this experiment:

•Troup	Jackson Co.	Paleudults soil
•Lakeland	Walton Co.	Quartzipsamments soil
•Leon	Duval Co.	Haplaquads soil
•Chipley	Jackson Co.	Quartzipsamments soil
•Norfolk	Alachua Co.	Paleudults soil

Blocking example 1

Graphical View



Introduce soil type as blocking factor

Example Sulfur Extraction in Soils

Soil	Solution	Sulfur
Troop	CaCl	5.07
Troop	NH4OAc	4.43
Troop	Ca(H2PO4)2	7.09
Troop	Water	4.48
Lakeland	CaCl	3.31
Lakeland	NH4OAc	2.74
Lakeland	Ca(H2PO4)2	2.32
Lakeland	Water	2.35
Leon	CaCl	2.54
Leon	NH4OAc	2.09
Leon	Ca(H2PO4)2	1.09
Leon	Water	2.70
Chipley	CaCl	2.34
Chipley	NH4OAc	2.07
Chipley	Ca(H2PO4)2	4.38
Chipley	Water	3.85
Norfolk	CaCl	4.71
Norfolk	NH4OAc	5.29
Norfolk	Ca(H2PO4)2	5.70
Norfolk	Water	4.98

	CaCl	NH4OAc	Ca(H2PO4)2	Water
Troop	5.07	4.43	7.09	4.48
Lakeland	3.31	2.74	2.32	2.35
Leon	2.54	2.09	1.09	2.70
Chipley	2.34	2.07	4.38	3.85
Norfolk	4.71	5.29	5.70	4.98

Example Sulfur Extraction in Soils

SAS program

SAS output

Blocking example 2

- The greenhouse tomato cropping virtual experimentation environment